STAT 2290 Exam 3D, April 2026, 9 problems, 75 minutes　　　　Name: ANSWERS

Read these instructions:

- Leaving the testing room results in a new exam given for unfinished problems.
- Three detached sheets of notes allowed. **Turn in these notes with your exam.**
- No electronics.
- You may leave answers in terms of combinations, permutations, factorials, exponentiation, $\times$, $\div$, $+$, $-$, $\sqrt{\bullet}$ of numbers. For instance, $10 \times \sqrt{5 \times 271}/17$ is an acceptable answer.

Useful critical values and truncated R outputs:

Confidence level c	Critical value z_c
90%	1.645
95%	1.96
99%	2.575

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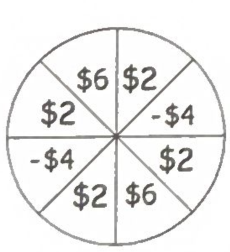
pnorm(-2) = 0.0227
pnorm(-1.5) = 0.0668
pnorm(-1) = 0.1586
pnorm(-0.5) = 0.3085
pnorm(0.5) = 0.6914
pnorm(1) = 0.8413
pnorm(1.5) = 0.9331
pnorm(2) = 0.9772
```

Problem 1. Estimate `pnorm(3, mean=6, sd=2)` from the above R outputs.

$= \text{pnorm}\left(\frac{3-6}{2}\right) = \text{pnorm}(-1.5) = 0.0668.$

Problem 2. You throw a dart, hitting the wheel below. You win or lose the monetary amount indicated by where your dart lands. Let X be the random variable modeling this monetary amount. Find the expected value and variance of X.



$E(X) = (2) \cdot \frac{1}{2} + (6) \cdot \frac{1}{4} + (-4) \cdot \frac{1}{4} = 1 + 1.5 - 1 = 1.5$

$E(X^2) = (2)^2 \cdot \frac{1}{2} + (6)^2 \cdot \frac{1}{4} + (-4)^2 \cdot \frac{1}{4} = 2 + 9 + 4 = 15$

$\sigma^2 = 15 - (1.5)^2.$

$X = x$	\$2	\$6	-\$4
$P(X=x)$	$\frac{1}{2}$	$\frac{2}{8}$	$\frac{2}{8}$

Problem 3. Rowan surveys an SRS of 100 students and find that 40 support a crackdown on underage drinking. Let $\hat{p}$ be the proportion of the sampled students who support cracking down on underage drinking.

(a) What is the mean of the sampling distribution of $\hat{p}$?

$$\mu_{\hat{p}} = \hat{p} = \frac{40}{100} = 0.4$$

(b) Find the standard deviation of the sampling distribution of $\hat{p}$. (Justify your answer.)

Check 10% Condition: There are $N \geq 10 \cdot 100 = 1000$ students at Rowan ✓

So $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.4 \cdot 0.6}{100}}$

(c) Is the sampling distribution of $\hat{p}$ approximately normal? (Justify your answer.)

Check large counts:

$n \cdot \hat{p} = 40 \geq 10$, $n \cdot (1-\hat{p}) = 60 \geq 10$.

So Yes, the samp. dist. of $\hat{p}$ is approx. normal.

Problem 4. The weights of a population of adult male monkeys is known to be normally distributed with a population mean of 20 lbs. and a population standard deviation of 2 lbs.

(a) What is the probability that a randomly selected monkey from this population weighs less than 21 lbs.?

$\text{pnorm}(21, \text{mean} = 20, \text{sd} = 2) = \text{pnorm}\left(\frac{21-20}{2}\right) = \text{pnorm}\left(\frac{1}{2}\right)$

$= 0.6914$.

(b) What is the probability that the average weight of a random sample of 4 monkeys from this population is less than 21 lbs.? (Justify your answer.)

n

- Check 10% Condition: there are $N \geq 10 \cdot 4 = 40$ adult male monkeys in population ✓
- Check ~~Large Counts: n~~ approx normality: pop. dist is normal ✓.
- Can proceed:
- $\text{pnorm}\left(21, \text{mean} = 20, \text{sd} = \frac{2}{\sqrt{4}}\right) = \text{pnorm}\left(\frac{21-20}{1}\right) =$

$= \text{pnorm}(1) = 0.8413$.

$(\approx 84\%)$

Problem 5. The weights of a population of adult penguins is known to be normally distributed with a population standard deviation of 2 lbs. Four penguins from this population weigh 30 lbs., 30 lbs., 31 lbs., and 33 lbs., respectively. From these information alone, determine a 95% confidence interval for the population mean.

- $\bar{x} = \frac{30+30+31+33}{4} = \frac{124}{4} = 31$

Check 10% Condition: There are $N \geq 10 \cdot 4 = 40$ adult penguins ✓

Check approx normality: pop. dist is already normal. ✓

- $c = 95\% \Rightarrow z_c = 1.96$
- $E = z_c \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{2}{\sqrt{4}} = 1.96$

So 95% CI is $31 - 1.96 < \mu < 31 + 1.96$

Problem 6. You wish to estimate, with 90% confidence, the population proportion of U.S. adults who eat fast food four to six times per week. Your estimate ~~must be~~ is accurate to within 3% of the population proportion.

$\boxed{\text{Find min. sample size so that}}$

- $c = 90\% \Rightarrow z_c = 1.645$
- $0.03 \geq E = z_c \cdot \sqrt{\frac{0.5 \cdot 0.5}{n}} = \frac{1.645 \cdot 0.5}{\sqrt{n}}$

$\Rightarrow \sqrt{n} \geq \frac{1.645 \cdot 0.5}{0.03}$

$\Rightarrow \boxed{n \geq \left(\frac{1.645 \cdot 0.5}{0.03}\right)^2}$

Problem 7. Let X be the random variable that records the number of "1"s you get when you roll a fair dice twelve times.

binomial RV with $n = 12$, $p = \frac{1}{6}$

(a) Determine the expected value of X.

$n \cdot p = 12 \cdot \frac{1}{6} = 2$

(b) Determine the standard deviation of X.

$\sqrt{np(1-p)} = \sqrt{12 \cdot \frac{1}{6} \cdot \frac{5}{6}}$

Problem 8. The time to failure (in months) of device is a random variable whose PDF is $f(x) = c/x^2$ if $x \geq 1$ and $f(x) = 0$ if $x < 1$.

(a) Find c.

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_1^{\infty} \frac{c}{x^2}\,dx = \int_1^{\infty} cx^{-2}\,dx = \left[\frac{cx^{-1}}{-1}\right]_1^{\infty} = [0] - \left[\frac{c(1)^{-1}}{-1}\right] = c$$

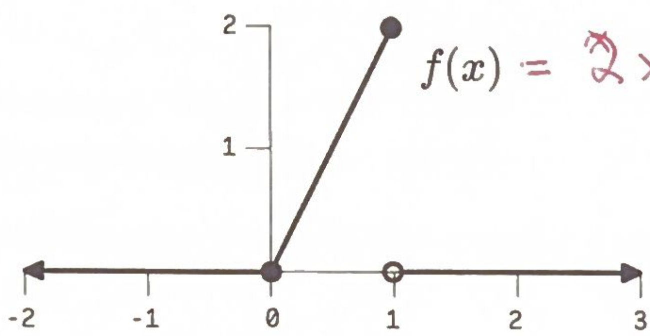
$$\Rightarrow \boxed{c=1}$$

(b) Find the probability that the device lasts more than 3 months.

$\boxed{c=1}$

$$P(X \geq 3) = \int_3^{\infty} f(x)\,dx = \left[\frac{cx^{-1}}{-1}\right]_3^{\infty} = [0] - \left[\frac{c(3)^{-1}}{-1}\right] = \frac{c}{3} = \frac{1}{3}$$

Problem 9. A random variable X has PDF $f(x)$ whose graph is shown:



$f(x) = 2x$ if $0 \leq x \leq 1$

(a) Find the CDF of X.

if $0 \leq x \leq 1$: $F(x) = \int_0^x f(x)\,dx = \int_0^x 2x\,dx = x^2 \Rightarrow F(x) = \begin{cases} 0 & x < 0 \\ x^2 & \text{if } 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$

(b) Find the expected value and variance of X.

$$E(X) = \int_0^1 x f(x)\,dx = \int_0^1 x \cdot 2x\,dx = \int_0^1 2x^2\,dx = \left[\frac{2x^3}{3}\right]_0^1 = \frac{2}{3}.$$

$$E(X^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 x^2 \cdot 2x\,dx = \int_0^1 2x^3\,dx = \left[\frac{x^4}{2}\right]_0^1 = \frac{1}{2}.$$

$$\Rightarrow \sigma^2 = E(X^2) - E(X)^2 = \boxed{\frac{1}{2} - \frac{4}{9}}.$$

Page	1	2	3	4
Points	15	20	30	20
Score				